Measuring market risk

Value at Risk

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1. **Understanding market risk**

In the light of the increased market instability of the recent years and the complex evolution in financial instruments, risk management tools have encountered a rapid and sophisticated development, with Value at Risk becoming one of the most prominent tools.

Market risk is one of the three major risk types, being generated by fluctuations in the market prices of instruments or market rates. The most important market risk factors are represented by currency, fixed-income, equity and commodities risk. While currency risk is straightforward and is generated by the specific volatilities and correlations between different currencies and movements in international interest rates, fixed-income risk is more comprehensive and can range from yield curve risk to basis risk, including specific risk factors like volatility risk or prepayment risk. Equity risk and commodities are associated with the corresponding changes in prices of equity instruments or portfolios and commodities.

At the level of financial institutions, market risk started to gain a more important role as a considerable part of the financial transactions moved away from the traditional banking operations and the regulatory bodies recognized its potential impact on the economic capital of the institutions. Therefore, starting with the 1996 Amendment of Basel Accord, market risk has been considered, along credit risk, in the assessment of the risk profiles of financial institutions and in the computation of their capital requirements, with Value at Risk being officially introduced as a “regulatory risk instrument”.

The recent financial crisis has raised, though, questions about the considerations underlying the isolated evaluation of market and credit risk. It seems that the two types of risks may have in certain situations compounding effect which were not considered in regulatory requirements and have led to an underestimation of their interaction. The separation of risk in the traditional categories is becoming more blurred and in fact, the large losses currently incurred in the trading books of investment banks were mostly attributed to the credit spread widening. Therefore, the traditional top-down approach in the organization-wide management risk of simply adding market with credit risk has to be reconsidered and transformed into a bottom-up approach of integrating the two types of risk.
2. An overview of Value at Risk

Value at Risk (VaR) is the maximum loss expected to be incurred at a given confidence level over a given horizon period. It is a statistical tool, based on the future profit and loss distribution of an instrument/portfolio, which leads to a probabilistic statement of the loss estimated to be exceeded in a certain number of situations (extreme events), but which does not tell us how much we could actually lose. The most frequently used VaR form is the relative one, where the potential maximum loss is reported relative to the expected profit and loss, taking also into account the time value of money.

The VaR computation is defined through the selection of its parameters, the time horizon and the confidence level, the choice being determined by a series of factors beyond the regulatory restrictions of 99% confidence level and 10 days horizon. Thus, the time period of VaR estimation should take into account the liquidity of the exposures on instruments, the frequency of trading activity within a portfolio and the underlying distribution of instruments values. The confidence level selection is more of a compromise between ensuring accuracy in the VaR estimation and detecting abnormality in the extreme regions of the distribution tails. Moreover, the risk aversion degree of the organization’s management and the cost of suffering a loss in excess of VaR determines the type of extreme events, for which capital coverage is desired and therefore influences the level of confidence.

VaR has gained popularity mainly due to its simplicity of showing in a single, easy to understand number the downside risk at organization-level, across different trading desks, businesses and markets. A frequently emphasized drawback of VaR in the literature consists in VaR not being a “coherent risk measure”\(^1\), since it does not present the subadditivity property, important for capturing the diversification effects. Still, this condition is fulfilled under the normality assumption of the distribution under analysis and cases where VaR fails to meet this property are usually rare. Therefore VaR remains a consistent risk measure, allowing a direct comparison between different trading exposures and implicitly facilitating the process of risk budgeting and risk limits establishment. Through these advantages, it achieves greater transparency for the risk profile of an institution and a superior strategic management.

\(^1\) As Artzner et al defines the concept. See: Return to RiskMetrics- The evolution of a standard, 2001
3. Value at Risk approaches

There are three approaches for computing VaR: the variance-covariance approach (delta normal approach), historical simulation and Monte Carlo simulation. The main difference between them is given by the underlying assumptions and models of generating P&L distributions. While for the variance-covariance VaR, the assumptions of normality and linearity in risk factors is crucial, since the computation is done with approximation of the changes in instruments/portfolios values, the other two approaches does not depend on a specific distribution, having the capability to accommodate VaR to different distribution types.

The normality hypothesis behind delta-normal VaR not only that it reduces the computation to a simple multiple of the standard deviation\(^2\), but allows also a marginal analysis of VaR by decomposing the risk in components. The RiskMetrics methodology refines the concept of normality, emphasizing that under a time-varying volatility the returns of the risk factors should follow a conditional normal distribution, while the observed distribution may be non-normal. A significant inconvenient of the normality assumption is the invalidation by the empirical distributions, many of which display “fat tails” characteristic leading to an underestimation of risk. In order to correct this weakness, alternatives to delta-normal have been developed, like using a t-distribution, a “normal mixture” distribution or a generalized-error distribution to better capture the empirical features of the distribution.

Nonetheless, the linearity assumption restricts also the application of delta-normal VaR to more complex portfolios with substantial options components or other instruments with a high degree of non-linearity. In this case, delta VaR has been extended to account for convexity/gamma effects, but it remains still an approximation that overlooks other risk factors and loses the easiness of normality.

Besides specifying the parameters of the distribution, delta VaR depends also on the volatility and correlation forecasts over the envisaged time horizon. This step is very important since it should reflect the volatility patterns over time, like clustering and mean-reversion and in the same time the changing correlation structures, both representing basic inputs in risk

\(^2\) Whereas the standard deviation is assumed to capture the entire uncertainty of the distribution.
management. Poor forecasts will fail to incorporate recent market news and trends and implicitly will not predict the future uncertainty of price movements, hindering reactive strategy of the investors towards shifting exposures to low or high volatility, given the risk aversion degree. Unfortunately, the traditional models of forecast, like exponentially moving average or Garch react too late, after a market shock has occurred instead of incorporating investors anticipations of changes in volatility and correlations.

Through historical simulation, the VaR estimate avoids the inconveniences mentioned above, since it makes no distribution assumption and no forecasts are needed. Historical VaR uses the returns observed in the past for the risk factors and projects them into the future, generating an entire P&L distribution out of which the VaR quintile is read off. Still, historical VaR has its own drawbacks, the major ones given by the dependence on the historical sample. Since this approach implies that the future is sufficiently like the past, it will fail to capture events that are possible to occur but did not occur in the past. In the same time, structural changes in the risk factors are underweighted in large historical samples, where to each information is given the same importance regardless of its age.

The most powerful VaR method is represented by Monte-Carlo, which like the historical simulation requires a full-valuation, but it enables us to simulate evolution paths for the risk factors according to the models that govern the price generation process. While Monte Carlo is a comprehensive approach which can fit any type of date, its complexity is also the one that makes it computationally intensive. Moreover, it is exposed to model risk, since the choice of the appropriate pricing models determines the result. For market risk factors, common used models are the Brownian motion for stock prices and foreign exchange, respectively equilibrium models and no-arbitrage models with all their alternatives for the yield movements, like the Vasicek or Cox-Ingersoll-Ross models. Despite the popularity of the mentioned models, they still are unable to capture some features of the distribution and simplify the pattern of price movements, ignoring thus the current structure of risk factors.

All these three approaches have in common that for more advanced instruments, their application is hindered by exposures to multiple risk factors. Therefore, an important step to implementation of the VaR methodology requires the mapping of instruments to “primitive risk factors” in order to reduce the dimensionality of data. The process of mapping involves selecting
the benchmark factors, for which data and forecasts are available and decomposing the exposures under analysis into positions on this factors - so called building blocks. VaR is then more easily computed at the level of each building block and at portfolio level, the positions can be aggregated across instruments. Considering these basic principles of mapping, it must be underlined that the process itself varies depending on the risk factors involved, from simple decomposing of stock exposures to more extended mapping of fixed-income instruments where the characteristics of the term structure of interest rates has to be considered.

The VaR analysis results provide valuable input in assessing the risk that a institution faces when trading on an instrument or portfolio, but more insight to the interactions taking place within a portfolio can be provided by assessing the risk contributions of different components to portfolio VaR. This can be done with instruments like Incremental VaR or Marginal VaR. While Marginal VaR tells us how the portfolio VaR changes if we enter into a new trade, selling or buying a specific position, IVaR is more useful, denoting the portfolio effect of relatively small increases or decreases in the holdings of the portfolio. The applicability of IVaR is more extensive since even less actively trading strategies require rebalancing the portfolio in certain market situations. The intuition behind these additional VaR measures is similar to the mean-variance framework, in that each component of the portfolio is correlated with the rest and therefore it has a risk relative to the portfolio, based on which the individual risk contribution to the overall risk is scaled. Baseline is that existing diversifications effects lead to different VaR contributions and implicitly a better selection of investments from a risk-return perspective.

4. Value at Risk on FX spot and forward

The theoretical aspects briefly presented in the previous section were applied on a foreign currencies portfolio, made out of two long positions on Euro and Dollar, each of 1 million. The main considerate in this choice of risk factors was the high volatility recorded by the foreign exchanges (FX) of the two currencies in the last year and the significant impact they had on the balance sheet of financial institutions. Therefore, the purpose of the assessment is to determine the potential maximum loss of this portfolio that can be estimated through the three approaches VaR and to compare between the results, emphasizing the implications of selecting just one of them in the risk management process. Finally, VaR capacity to capture hedging potential in
different risk exposures is presented through a forward contract, combined with one of the long position in the euro-dollar portfolio.

VaR is computed for a 1 day horizon, at a 95% confidence level and the sample data is collected daily between January 2007- April 2009, counting 575 observations. For determining VaR through the variance–covariance approach, the volatilities and correlations of Euro and Usd are forecasted for 15 April using the EWMA methodology according to RiskMetrics. Since there are only two risk factors, my forecasts are constructed based on individual decay-factors for the time series and not on the general ones used by RiskMetrics. There are obtained based on the assumption that the daily mean return at the population level is 0, which eliminates the possible distortions of an inaccurate expected return and on the minimization function of the root mean squared error of the forecasts relative to the ex-post records.

The volatility and correlations obtained differ from the sample descriptive statistics and are consistent with the idea behind the EWMA that recent market developments should be over weighted- in this case their level is influenced by the volatility and correlation pattern of the last ca. 40 days of trading, corresponding to the period February-April 2009. Therefore, the WMA forecasts capture the mean reverting trend of volatility after the unusual levels recorded in the autumn of 2008, at the height of the financial crisis, and in the case of the dollar remains clustered around higher values due to the financial events on international markets. What concerns the correlation, the EWMA forecast is much lower then the historical estimate, allowing therefore a higher diversification effect at the portfolio level.

Under the assumption of normality, the parametric VaR for the individual FX positions confirm the higher risk inherent in the dollar, but also a reduction in the market risk due to diversification, of ca. 15% of the undiversified VaR:

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<tr>
<th></th>
<th>EUR</th>
<th>USD</th>
</tr>
</thead>
<tbody>
<tr>
<td>VaR individual</td>
<td>30,564.87</td>
<td>56,756.81</td>
</tr>
<tr>
<td>Undiversified VaR</td>
<td>87,321.68</td>
<td></td>
</tr>
<tr>
<td>Diversified VaR</td>
<td>74,589.93</td>
<td></td>
</tr>
<tr>
<td>CVaR</td>
<td>21,963.64</td>
<td>52,626.29</td>
</tr>
</tbody>
</table>

In the same time, it can be observed that the higher relative risk of the dollar it is translated also in a higher contribution to the portfolio VaR, the sensitivity of VaR to changes in this position being higher and leading implicitly to a tighter monitoring of the exposure value
relative to Usd.

Due to the importance of the normality hypothesis, the individual return distributions and also the portfolio one are tested for this assumption. At the level of empirical observations, they display excess kurtosis, drawing attention on the potential presence of “the fat tails” feature and an underestimation of the risk. But, if we test according to the RiskMetrics assumption of conditional normality, the result tend to validate the portfolio parametric VaR because at the portfolio level the Jarque-Bera test does not reject at a 95% confidence level the null hypothesis of normality.3

Conducting a historical simulation, we observe that the portfolio VaR is not consistent with the implied leptokurtosis of the empirical distribution. The diversified absolute VaR amounts to 65,590 Ron, ca. 12% lower then the variance-covariance VaR. In the same time, the higher historical correlation reduces the diversification potential from 15% to 9% and diminishes the differences between the CVaR of the two currencies. The lower historical VaR under the circumstances of a higher historical portfolio standard deviation may be explained by the left tail of the empirical distribution, which seems to be flatter then the normal one, leading to lower extreme losses.

The Monte Carlo simulation is conducted based on a Brownian motion model of changes in the foreign exchanges, which again leads to a normal P&L distribution. Using a Cholesky decomposition to reconstruct the correlation pattern between the random variables lying at foundation of the returns and the EWMA forecast used in the parametric approach, 5,000 return scenarios are generated, leading to a VaR of 72,813.06 Ron, very close to the parametric VaR. The fact that two values does not perfectly match although assuming a normal distribution is generated by the sampling variability inherent in any sample based methods. The construction of confidence intervals around the VaR estimates concludes that Monte Carlo VaR is within two standard error from the parametric VaR estimate.

After completing the three VaR computations, the inclination is to accept the implementation of a variance-covariance VaR and in this aspect a validation test is conducted at the level of 121 VaR estimates, to find out in how many situations the maximum loss estimated

3 With the comment, thought, that the tests are conducted on a sample of just 121 returns standardized by the corresponding standard deviation forecasts.
is exceeded by the actual loss. In our sample, this is the case of 6 days out of 121, meaning 5% exceptions. Even if this failure rate is consistent with the predicted 5% exceptions implied by the 95% confidence level, we still test this failure rate by applying a Kupiec test. The value obtained for the likelihood ratio falls within the acceptance region and therefore the parametric VaR method cannot be invalidated.

In light of the losses measured by VaR for the exposure at currency risk, the combination of a spot long position with a short forward position on 1 million euro is analyzed in order to determine the VaR reduction potential of this strategy. The initial value of the forward contract is set to the theoretical one, under the validation of interest rate parity. For our purpose the forward VaR is computed based on the mapping of this instrument to three risk factors: the Ron/Eur FX, the 3 months Euribor rate and the 3 months Robor rate. Thus, the forward is replaced by a portfolio made out of a short position on euro, a short position on a 3 months Euro-denominated zero-coupon bond with a face value of 1 million euro and a long position on a Ron-denominated zero coupon bond with a face value of the forward notional amount of 4,271,152,54 Ron. The P&L distribution of the forward contract is determined based on this three building blocks and the corresponding fluctuations in the risk factors. The present value of the components represents practically the current exposures to risk.

VaR parametric is constructed in the same manner as the euro-dollar portfolio mentioning that in this case the interest rate risk is expressed as the risk in the discount factors - the risk given by the “bond” prices. The EWMA forecasts are thus determined for the price volatilities and correlations and not for the yields. The results obtained are here even lower then the historical volatilities while the correlation pattern displays opposite signs then the historical one. The Euribor and Robor prices are low positively correlated leading to a small diversification potential, while euro FX is negatively correlated with both of them, drawing attention on the potential hedging effect of these correlations. Nonetheless the arguments brought for EWMA forecasts in case of the euro-dollars portfolio are also here valid, capturing the most recent trends in volatility and correlation.

The diversified VaR for the forward amounts to 31,262 Ron, more then 90% out of which is contributed by the FX with a CVaR of 30,363 Ron. This shows that the driving force behind

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4 The terms “Euribor” and “Robor” are used in this case
the forward price is the currency risk and not the money market risk, fairly expected given the very small daily changes in the interest rates:

<table>
<thead>
<tr>
<th></th>
<th>FX</th>
<th>Euribor</th>
<th>Robor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diversified VaR</td>
<td>31,262</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beta</td>
<td>0.9712</td>
<td>-0.0013</td>
<td>-0.03</td>
</tr>
<tr>
<td>CVaR</td>
<td>30,363.04</td>
<td>-39.61</td>
<td>939.43</td>
</tr>
</tbody>
</table>

The analysis at CVaR level confirms the existence of a hedging position, in our case through the Euribor exposure. The fact that the Fx is negatively correlated with Euribor and they have same sign positions determinates the Euribor fluctuations to diminish the portfolio VaR. Unfortunately the effect is rather small, due to the low volatility. On the other side, Robor correlation is also

The historical simulation yields in this case a VaR estimate which is almost 45% higher then the parametric one, amounting to ca. 45,058 Ron. The risk contributions of the components is also affected by historical correlations, leading this time to a Robor hedging position, which compensates not only the entire interest rate risk but also a part of the currency exposure. The difference in the potential maximum loss of the two methods is even more significant then the different results obtained for the spot portfolio, casting doubt on the parametric approach. The leptokurtosis of the distribution comes to support the results, but the Jarque-Bera test conducted on the forward returns does not reject the normality assumption, leading to controversial arguments for the two methods. Moreover, if the differences between historical volatilities and EWMA forecasts are removed, the VaR gap diminishes to only 8%, not sufficient to reject the parametric VaR. On the other side, the fact that the actual loss incurred (36,353 Ron) on 15 April, exceeds the parametric VaR with 16% and that CvaR’s are not consistent with the forecast (no hedging effect obtained) undermines the use of the simple VaR method.

The Monte Carlo simulation with simplifing hypothesis for the risk factors movements, conducted on 10 iterations, obatins a set of VaR ranging from 31,036 to 32,094 Ron, in line with the delta-normal VaR.

In the forward-spot combination, the exposures on euro offset each other almost entirely, leaving unhedged less then 0.5% of the initial value of the long exposure on euro. This time, with parametric VaR, the hedging effects of Euribor and Euro contributes to this risk elimination,
achieving thus a 92% reduction in the risk of the forward and that of the long spot position, respectively. Therefore, the only risk remaining in the portfolio amounting to 2.580 Ron comes entirely from the Robor fluctuations.

Conclusions

Computing VaR for an euro-dollar portfolio underlines the importance of the currency risk exposure and the potential of diversification across international markets. Moreover the marginal analysis applied on portfolio VaR offers valuable information on the currencies contributing the most to the risk profile of the financial institution, in our case the dollar and makes possible the evaluation of the impact in changing dollar exposures on the total risk.

The implementation of the three approaches emphasizes the different results obtained and therefore the need to determine which one is the most reliable. The backtesting of the delta-normal VaR and the positive results of the normality tests provide important arguments in favor of this approach, also in the light of an investor with higher risk aversion.

The currency risk can be hedged through term contracts, like forwards, the VaR approach capturing this risk reduction. The decomposition of the forward in the basic building blocks underlines once more the overweight of the currency risk relative to the money market risk on very short time horizon and reflects the impact of correlation pattern on the total risk. Both, parametric approach and historical simulation, display hedging effect from the interest rate components, but the results are different, with a higher VaR distorted by the past, signaling its weakness. Therefore, in this case a risk management strategy combining the updated parametric VaR with more conservative historical VaR should provide a higher degree of prudence, while not allocating inefficiently capital.

Thus, combining spot positions with forwards on foreign exchange allows a financial institution to better face risk requirements, remaining only with open money market position, which except for very extreme events should not jeopardize the institution’s profile on short-term.
Bibliography:

- www.bnro.ro; www.euribor.org

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